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Animal science application of robust tests: effect of zeolite and initial weight on fattening performance of cattle

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A literature search shows that robust techniques are rarely used in the animal sciences. Traditionally, normal theory tests are used to determine whether there are significant differences among group means. However, normal theory tests are optimal only if the distribution of error terms is normal and in practice, non-normal distributions are more prevalent. Therefore, in this study, it is aimed to present robust tests based on modified maximum likelihood (MML) estimators for testing the equality of the group means. A real data set about the effect of zeolite and initial weight on feedlot performance of Brown Swiss cattle is the subject of the study. Results showed that the test statistics presented in this study are more powerful than the traditional normal-theory tests. They are also more robust against departures from assumed distribution and outliers.

KEY WORDS: Generalized logistic / non-normality / outliers / modified maximum likelihood / least squares / robustness

Classical statistical procedures assume that the errors are distributed normally. However, the normality assumption is not valid in most practical situations [e.g. Pearson 1932, Geary 1947, Huber 1981, Tiku *et al*. 1986, Tan and Tiku 1999, Şenoglu and Tiku 2001, 2002, 2004]. The normal-theory statistics loose their optimality under departures from normality. One method of analysing non-normal data is to invoke Box-Cox [1964] normalizing transformation. It should be clear that not all data can be amenable to this transformation or, for that matter, any other so called normalizing transformation. Moreover, the location and scale parametres may not translate into

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meaningful parametres after data transformation. To overcome these difficulties, in the analysis of the data, original data rather then the transformed data will be used. It should be noted here that law of large numbers and the central limit theorem can work effectively only if the number of experimental units are very large. However, the number of experimental units are not very large in the context of experimental design and therefore, the statistical analyses based on normal theory are not reliable when the distribution of the error terms is not normal.

The presence of outliers is another indication for non-normality. In fact, Huber [1981] stated that, as a rule, samples contain 5 to 10% outliers defined as extreme observations relative to the rest of the sample or large measurement errors. Outliers can have a disproportionately large impact on the statistical analyses. They will decrease the efficiency of least squares (LS) estimators and therefore the power of the normal theory t and F tests.

Another method for analysing non-normal data is the well-known nonparametric test based on ranks. It does not make the usual distributional assumptions of the normal theory tests and are nearly as powerful as the classical tests even if the underlying distribution of error terms is normal. It is more powerful and robust than the classical tests when the samples strongly deviate from normality.

In this paper, robust t and F test developed by Şenoglu [2000] and Şenoglu and Tiku [2001, 2002] for testing the linear contrasts and the equality of treatment means are presented, respectively. These tests are based on MML estimators and apply to the data directly rather than the transformed data or ranks of the data. Their tests are also quite robust to non-normality and outliers besides being more powerful than the classical normal-theory t and F tests.

They considered Weibull *W(p,* σ) and Generalized Logistic *GL(b,* σ*)* as an error distribution. However, in this paper, the results for *GL(b,* σ*) (b>)* distribution whose probability density function (pdf) is given by

$$GL(b,\sigma) = \frac{b}{\sigma}\frac{\exp(-e/\sigma)}{[1+\exp(-e/\sigma)]^{b+1}}, \ (-\infty < e < \infty). \qquad (1.1)$$

will be presented,
where *e* represents the error terms.

The data almost always can be modeled with one of the symmetric or skew distributions and the generalized logistic distribution represents a very wide range of skewed distributions from negatively skewed (*b<1*) to positively skewed (*b>1*) distributions. For, it is the well-known logistic distribution which is a plausible alternative to the normal distribution. This flexibility is the reason for considering (*GL(b,* σ) as an error distribution.

Materials and methods

Material of the study included 40 Brown Swiss cattle and study was conducted at Suleyman Demirel University, Atabey Vocational High School in February 2002, in Turkey. In the beginning of the study in order to alleviate the transportation stress and make calves accustomed to their environment, cattle were given hay in the feedlot. After cattle were accustomed to the feedlot they were weighed. Twenty heaviest cattle with mean weight of 207 kg were allotted into two lots and similarly 20 lighter cattle with mean weight of 160 kg were allotted into two lots and thus each lot consisted of 10 cattle. Treatments were; 1- one paddock of cattle in heavy group and one paddock of cattle in lighter group both receiving 1% zeolite during the experiment (ZE), 2- one paddock of cattle in heavy group and one paddock of cattle in lighter group not receiving zeolite during the experiment (control). During the experiment one cattle in each control group was removed due to health concerns. The feedlot facility consisted of open lots with 2 meters of apron and 5% slope to the north. Animals were fed in fence-line concrete bunks, providing 90 cm feedbunk space per animal, on the south side of the lot and one automatic waterer was shared between two pens. Cattle were fed *ad libitum* and feed intake levels were provided such that feed always was available in the feedbunks. The ration was formulated according to NRC [1996] recommendations. Sugarbeet pulp, sugarbeet molasses, grass hay, vetch were used as roughage source whereas barley, corn, cottonseed meal, urea, vitamin-mineral premix were used as concentrates. Cattle were weighed every two weeks individually and average daily gain for that period and throughout the experiment were calculated. Animals were fed for 292 days and mean daily gain was calculated as final weight

Table 1. Descriptive statistics of variables by treatments*

Treatment	Minimum	Maximum	Mean	Standard error	Treatment variance
ZE					
mean daily gain (kg)	0.72	1.14	0.90	0.028	0.00078
feed efficiency	6.49	11.15	8.58	0.26	0.068
dressing (%)	50.38	58.68	54.99	0.49	0.24
ınitial weight (kg)	127	273	184	7.45	55.50
final weight (kg)	357	605	446	13.00	169
carcass weight (kg)	183	355	246	8.73	76.21
Control					
mean daily gain (kg)	0.69	1.21	0.86	0.029	0.00084
feed efficiency	6.53	10.63	8.82	0.26	0.068
dressing (%)	52.97	58.60	55.72	0.39	0.15
ınitial weight (kg)	124	247	184	6.87	47.20
final weight (kg)	353	600	436	13.59	184.69
carcass weight (kg)	187	336	243	8.34	69.56

*Skewness and kurtosis values of the error terms for each variable are very close to those of $GL(b=1.0,\sigma)$. Therefore they are not reported for sake of brevity.

minus initial weight divided by days on feed. After animals were slaughtered hot carcass weight was obtained and thus dressing percentage was calculated. Descriptive statistics for treatmets are provided in Table 1.

The Modified Maximum Likelihood (MML) methodology was introduced by Tiku [1967]. In the context of experimental design, Şenoglu and Tiku [2001, 2002] extended this method to analysis of variance with non-normal error distributions and to linear contrasts with non-identical error distributions. Those papers provided substantial improvement in analysing the data sets showing violations of the fundamental assumptions of the traditional statistical inference such as:

i) the distribution of the error terms are normal;

ii) the error distributions from treatment to treatment have constant variance;

The MML method works as follows:

1. For the linear model workout the likelihood functions $\partial \ln L/\partial \mu$, $\partial \ln L/\partial \tau$, etc.
2. Express these equations in terms of order statistics. Since complete sums are invariant to ordering, *i.e.*,

$$\sum_{i=1}^{n} f(y_i) = \sum_{i=1}^{n} f(y_{(i)}),$$

where $f(y)$ is any function of y.

3. Approximate intractable function by linear functions using the first two terms of Taylor series expansions around

$$E(z_{(i)}), 1 \leq i \leq n \text{ where } z_{(i)} = \frac{y_{(i)} - E(y_i)}{\sigma}$$

4. The solutions of the resulting equations are the MML estimators.

The MML estimators have the following very attractive properties:

1. The MML estimators are known to be asymptotically efficient.
2. The MML estimators are essentially as efficient as the MLE and the two are numerically the same (almost) even for small samples.
3. The MML estimators have the explicit solutions and have the same structure irrespective of the underlying distribution.
4. The MML estimators have the invariance property.
5. The MML estimators are robust. Because, they are fully efficient (or nearly so) under the assumed distribution and maintain high efficiency under the plausible alternatives to the underlying distribution.

It should also be noted that MML methodology can be used for any location-scale distribution of the type $(1/\sigma)\, f\, [(y - \mu/\sigma]$.

Hypothesis testing is a very important problem in statistical theory and practice. In this section, a modified versions of the robust t test for linear contrasts and two-way ANOVA F test developed by Şenoglu [2000] and Şenoglu and Tiku [2001, 2002] are presented. They showed that their statistics are robust and more powerful than the normal-theory-based statistics and are insensitive to outliers. Since their tests are based on MML estimators which have high efficiency not only under the assumed distribution but also under reasonable alternatives.

Robust Linear Contrasts: since in experimental designs estimating linear contrasts of k means is of main interest.

Consider the model

$y_{ij} = \mu_i + e_{ij}$, *(i=1,2,...k; j=1,2,...n)*

where e_{ij} are iid (identically and independently distributed) and have a Generalized Logistic distribution $GL(b, \sigma)$.

Under this model to test the null hypothesis

$$H_0 : \sum_{i=1}^{k} l_i \mu_i = 0 \qquad \text{where } l_1 + l_2 + \dots l_k = 0),$$

they define the statistic [Şenoglu, 2000]

$$t_{MML} = \frac{\sum_{i=1}^{k} l_i \hat{\mu}_{i.}}{\hat{\sigma}\sqrt{\sum_{i=1}^{k} l_i^2 \frac{1}{m_i(b+1)}}} \tag{3.1.1}$$

where

$$\begin{aligned}
&\hat{\mu}_i = \hat{\mu}_{i.} + (\Delta/\mathrm{m})\hat{\sigma},\ \sigma = \left(B + \sqrt{B^2 + 4NC}\right) \Big/ 2\sqrt{N(N-k)},\ N = k \times n \\
&\hat{\mu}_{i.} = (1/m)\sum_{i=1}^{n} \beta_i y_{(i)},\ \alpha_i = \left(1 + e^t + te^t\right) \Big/ \left(1 + e^t\right)^2,\ \beta_i = e^t \Big/ \left(1 + e^t\right)^2, \\
&t = t_{(i)} = -\ln\left(q_i^{-1/b} - 1\right),\ q_i = i/(n+1), \\
&\Delta_i = (b+1)^{-1} - \alpha_i,\ m = \sum_{i=1}^{n} \beta_i,\ \Delta = \sum_{i=1}^{n} \Delta_i, \\
&B = (b+1)\sum_{i=1}^{n} \Delta_i \left(y_{(i)} - \hat{\mu}_{i.}\right),\ C = (b+1)\sum_{i=1}^{n} \beta_i \left(y_{(i)} - \hat{\mu}_{i.}\right)^2.
\end{aligned} \tag{3.1.2}$$

The asymptotic distribution of t^* is Normal $N(0.1)$. For small n, the null distribution of t_{MML} is closely approximated by a Student t distribution with $\upsilon = \sum_{i=1}^{k} n_i - k$ degrees of freedom.

It should be realized that t_{MML} is the one-sample and two-sample test for $k = 1$ and $k = 2$, respectively.

Robust Two-Way ANOVA F Test. The problem of estimating and testing the equality of block means, treatment means and interaction is of vital importance in statistical theory and practice. To see the estimators of the parametres and test statistics, consider the model

$$y_{ijl} = \mu + \gamma_i + \delta_j + \tau_{ij} + e_{ijl}, \left(i = 1,2,\dots,k;\ j = 1,2,\dots,c;\ l = 1,2,\dots,n\right) \tag{3.2.1}$$

where μ is the overall mean, γ_i is the fixed effect of i-th treatment, δ_j is the effect due to *j*th block, τ_{ij} is the interaction effect and e_{ijl} is the random errors. Without loss of generality it is assumed that

$$\sum_{i=1}^{k}\gamma_i = \sum_{j=1}^{c}\delta_j = \sum_{i=1}^{k}\tau_{ij} = \sum_{j=1}^{c}\tau_{ij} = 0\,.$$

To test the following null hypotheses

$$\begin{aligned} &H_{01}:\gamma_1=\gamma_2=\dots\dots\dots=\gamma_k=0\\ &H_{02}:\delta_1=\delta_2=\dots\dots\dots=\delta_c=0\ \text{ and}\\ &H_{03}:\tau_{11}=\dots\dots=\tau_{kc}=0,\end{aligned} \tag{3.2.2}$$

the following test statistics are defined [Şenoglu and Tiku 2001].

$$F_1=\frac{(b+1)\sum_{i=1}^{k}\left(\sum_{j=1}^{c}m_{ij}\right)\hat{\gamma}_i^2}{\hat{\sigma}^2},\ F_2=\frac{(b+1)\sum_{j=1}^{c}\left(\sum_{i=1}^{k}m_{ij}\right)\hat{\delta}_j^2}{\hat{\sigma}^2},\ F_3=\frac{(b+1)\sum_{i=1}^{k}\sum_{j=1}^{c}m_{ij}\hat{\tau}_{ij}^2}{\hat{\sigma}^2}\,. \tag{3.2.3}$$

respectively.

The MML estimators of the parametres are

$$\hat{\mu}=\hat{\mu}_{...}+(\Delta/\sum_{i=1}^{k}\sum_{j=1}^{c}m_{ij}),\ \hat{\gamma}_i=\hat{\mu}_{i..}-\hat{\mu}_{...},\ \hat{\delta}_j=\hat{\mu}_{.j.}-\hat{\mu}_{...},\ \hat{\tau}_{ij}=\hat{\mu}_{ij.}-\hat{\mu}_{i..}-\hat{\mu}_{.j.}+\hat{\mu}_{...},\ \hat{\sigma}=\frac{B+\sqrt{B^2+4NC}}{2\sqrt{N(N-kc)}},$$

where

$$\hat{\mu}_{i..}=\frac{\sum_{j=1}^{c}m_{ij}\hat{\mu}_{ij.}}{\sum_{j=1}^{c}m_{ij}},\ \hat{\mu}_{.j.}=\frac{\sum_{i=1}^{k}m_{ij}\hat{\mu}_{ij.}}{\sum_{i=1}^{k}m_{ij}},\ \hat{\mu}_{ij.}=\frac{\sum_{l=1}^{n_i}\beta_{ijl}y_{ij(l)}}{m_{ij}},\ \hat{\mu}_{...}=\frac{\sum_{i=1}^{k}\sum_{j=1}^{c}\left(\sum_{l=1}^{n_i}\beta_{ijl}y_{ij(l)}\right)}{\sum_{i=1}^{k}\sum_{j=1}^{c}m_{ij}}$$

$$\beta_{ijl}=e^t/\left(1+e^t\right)^2,\ \alpha_{ijl}=1+e^t+te^t/\left(1+e^t\right)^2,\ t=t_{ij(l)}=-\ln((l/(n+1))^{-1/b}-1) \tag{3.2.4}$$

$$m_{ij}=\sum_{l=1}^{n}\beta_{ijl},\ \Delta_{ijl}=(1+b)^{-1}-\alpha_l,\ \Delta=\sum_{l=1}^{n_i}\Delta_{ijl},$$

$$B=(b+1)\sum_{i=1}^{k}\sum_{j=1}^{c}\sum_{l=1}^{n_i}\Delta_{ijl}\left(y_{ij(l)}-\hat{\mu}_{ij.}\right),\ C=(b+1)\sum_{i=1}^{k}\sum_{j=1}^{c}\sum_{l=1}^{n_i}\beta_{ijl}\left(y_{ij(l)}-\hat{\mu}_{ij.}\right)^2.$$

For large n, their null distributions are central F with degrees of freedom (ν_1, ν_4), (ν_2, ν_4) and (ν_3, ν_4), respectively:

$\nu_1 = k - 1$, $\nu_2 = c - 1$, $\nu_3 = (k - 1)(c - 1)$ and $\nu_4 = kc(n - 1)$

Even for small n, these *F* distributions provide satisfactory approximations to the percentage points.

The estimators and the test statistic formulas for the one-way classification model are similar. Therefore, for the sake of simplicity they are not reproduced. However, the detailed information about the robust one-way ANOVA can be seen from Şenoglu [2000] and Şenoglu and Tiku [2001].

Results and discussion

The Normal Q-Q plots of the observations indicate that the distribution of the error terms is not normal. Because, the quantile pairs do not follow a straight line pattern. Therefore, Q-Q plots as in Şenoglu and Tiku [2001, 2002] are constructed and concluded that $GL(b = 1.0, \sigma)$ is the most appropriate distribution for the error terms for all variables. Table 2 shows the standard errors of the differences of control and zeolite groups and Table 3 shows the F values for various variables used in the experiment.

Table 2. Standard errors and t statistic

Trait	$SE(\hat{\mu}_1 - \hat{\mu}_2)$	t_{MML}	$SE(\bar{x}_1 - \bar{x}_2)$	t_{LS}
Average daily gain	0.038	-1.0407	0.040	-0.857
Feed efficiency	0.373	0.6428	0.374	0.642
Dressing	0.620	1.2248	0.643	1.136
Initial weight	9.741	0.2146	10.211	-0.019
Final weight	17.385	-0.6552	18.830	-0.542
Carcass weight	11.339	-0.2461	12.164	-0.232

Degrees of freedom for both t_{MML} and t_{LS} are $\upsilon = \sum_{i=1}^{k} n_i - k$.

The MML method and the LS method give essentially the same results (except for the initial weight effect in dressing percentage). In other words, the tests based on the MML estimators and the LS estimators are in accordance in rejecting or not rejecting the null hypothesis. However, the test statistics based on the MML estimators have the smaller scale estimates. This clearly indicates the superiority of the present method. Also, using the method presented in this study has the clear practical advantages since an assumed model is hardly ever exactly correct. The MML method, however, is robust to numerous departures from the assumed model and to the presence of outliers. Moreover, the MML method is computationally quite straightforward besides being easy to compute.

The test statistics presented in this study are more powerful than the traditional normal-theory tests. They are also more robust against departures from assumed distribution and outliers. These results are also supported by the simulation studies conducted by Şenoglu and Tiku [2001, 2002]. The robustness of the present method is due to the fact that the method gives small weights to the observations in the direction of the long tail(s) and this depletes the dominant effect of the outliers. For n=18, for example, following values are obtained:

Table 3. ANOVA based on the generalized logistic $GL(1.0, \sigma)$

Average daily gain (kg)			**Feed efficiency**		
Source	F_{MML}	F_{LS}	Source	F_{MML}	F_{LS}
Initial weight	1.400	2.018	Initial weight	1.730	1.285
Treatment	0.647	0.739	Treatment	0.320	0.407
Interaction	0.070	0.315	Interaction	0.024	0.200
Scale estimate	$\hat{\sigma} = 0.074$	$\tilde{\sigma} = 0.122$	Scale estimate	$\hat{\sigma} = 0.694$	$\tilde{\sigma} = 1.148$
Dressing (%)			**Initial weight (kg)**		
Source	F_{MML}	F_{LS}	Source	F_{MML}	F_{LS}
Initial weight	3.551*	2.913ns	Initial weight	46.155**	48.395**
Treatment	1.119	1.331	Treatment	0.015	0.001
Interaction	0.035	0.122	Interaction	0.117	0.143
Scale estimate	$\hat{\sigma} = 1.136$	$\tilde{\sigma} = 1.928$	Scale estimate	$\hat{\sigma} = 11.922$	$\tilde{\sigma} = 20.689$
Final weight (kg)			**Carcass weight (kg)**		
Source	F_{MML}	F_{LS}	Source	F_{MML}	F_{LS}
Initial weight	13.333**	15.647**	Initial weight	12.988**	14.469**
Treatment	0.444	0.406	Treatment	0.085	0.073
Interaction	0.041	0.065	Interaction	0.021	0.013
Scale estimate	$\hat{\sigma} = 28.882$	$\tilde{\sigma} = 49.310$	Scale estimate	$\hat{\sigma} = 18.830$	$\tilde{\sigma} = 32.205$

Degrees of freedom of initial weight, treatment and interaction for both F_{MML} and F_{LS} are (υ_1, υ_4), (υ_2, υ_4) and (υ_3, υ_4) respectively. Here $\upsilon_1 = k-1$, $\upsilon_2 = c-1$, $\upsilon_3 = (k-1)(c-1)$ and $\upsilon_4 = kc(n-1)$.

*Significant at p≤0.05..

**Significant at p≤0.01.

$b = 1.0$ β_i = 0.050 0.094 0.133 0.166 0.194 0.216 0.233 0.244 0.249
0.249 0.244 0.233 0.216 0.194 0.166 0.133 0.094 0.050.

Conclusion

Summarazing, the parametric t and F tests are optimal only if the error distribution is normal. However, if there is a doubt about the normality of the error terms or there exist outliers in the data set, the use of robust methods in analyzing real data sets is recommended.

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Zastosowanie testów odpornościowych w naukach o zwierzętach: wpływ zeolitu i masy początkowej na opas bydła

S t r e s z c z e n i e

Według dostępnego piśmiennictwa, w naukach o zwierzętach rzadko stosowane są adekwatne metody, kiedy mamy do czynienia z rozkładami nienormalnymi. Do oceny różnic między średnimi grup wykorzystywane są testy zakładajace normalność rozkładu. Testy te spełnaja swoją rolę tylko wówczas, gdy rozkład błędów jest normalny, podczas gdy w praktyce dominują rozkłady nienormalne. Stąd też celem pracy jest przedstawienie metody opartej na zmodyfikowanej metodzie największej wiarygodności (MML) optymalizującej moc dopasowania testu przy porównaniu średnich grup. Rzeczywisty zestaw danych dotyczących wpływu zastosowania zeolitu i masy początkowej opasu bydła Brown Swiss posłużył do porównania metod. Wyniki wykazały, że testy zaproponowane w pracy mają większą moc niż testy normalne. Wykazują one większą moc także wtedy, gdy rozkłady odbiegają od normalnych lub gdy poszczególne obserwacje znacznie odbiegają wartością od innych.