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Factors affecting the shape of lactation curves in Polish Holstein-Friesian cows*

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The objectives of this study were (1) to examine different shapes of lactation curves when the Wilmink function is fitted to test-day yields, and (2) to analyze the relationships between lactation curve shape and factors determining it in Polish Holstein-Friesian (HF) cows. Test-day milk yields were collected for 1,359,040 Polish Holstein-Friesian cows that calved from 1995 to 2008. A multiple-trait prediction (MTP) procedure was used to fit individual lactation curves and estimate partial, peak and 305-d lactation yields using the Wilmink function.

The Wilmink function was able to model two groups of lactation curves: standard and atypical. Atypical curves were found more often in later lactations rather than first lactations of Polish HF cows. For standard curves, peak yields (on average, about 23 kg in the first and 28-30 kg in later parities) occurred at around 38 days in milk (DIM) in the first lactations and 30 DIM in later lactations. For atypical curves, mean milk yields at peak were similar in value.

Milk yield recording should start as early as possible, because the distance between the first test and calving is the factor that significantly affected the occurrence of atypical lactation curves. An effect of season and parity on the occurrence of atypical curves also was observed in the Polish Holstein-Friesian population. The effect of season was especially noticeable for cows that calved between April and September with only one yield recorded.

KEY WORDS: dairy cattle / lactation curve shape / milk yield / Wilmink model

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Modelling of milk yield during the lactation period is one of the most successful applications of mathematical functions in agriculture. Tools that can mimic the milk production process are useful to physiologists, nutritionists and geneticists [Macciotta *et al.* 2011]. Models able to predict future milk yields provide valuable information applicable in breeding and management decisions, and are of value for many practical purposes such as health monitoring, individual feeding or genetic evaluation [Macciotta *et al.* 2005]. An appropriate model can describe the shape of the lactation curve of a particular cow and then predict values such as total or 305-d milk yields, peak milk yield, time to peak, or persistency of lactation [Dematawewa *et al.* 2007, Lopez *et al.* 2015].

Many studies have addressed lactation curve modeling and have compared goodness of fit for different mathematical functions [Olori *et al.* 1999, Druet *et al.* 2003, Macciotta *et al.* 2005, Silvestre *et al.* 2006, Dematawewa *et al.* 2007, Otwinowska-Mindur *et al.* 2013]. A typical lactation curve has an ascending phase to a peak (in the first 30 to 50 days post partum), followed by a descending phase (until the cow is dried off). Several models have been proposed to describe the standard lactation curve pattern for milk production. Early applications used parametric functions, such as those of Wood [1967], Wilmink [1987] or Ali and Schaeffer [1987]. As a possible alternative to parametric models, general functions such as Legendre orthogonal polynomials or splines have been proposed. Although they have a larger number of parameters, their advantage is connected with their remarkable capacity to fit a great range of shapes [Silvestre *et al.* 2006, Macciotta *et al.* 2011]. Olori *et al.* [1999], Silvestre *et al.* [2006] and Otwinowska-Mindur *et al.* [2013] reported that a larger number of parameters in a model gives a better fit to the data, while models containing fewer parameters, often with a biological interpretation (e.g. Wilmink), are also recommended. Silvestre *et al.* [2006] concluded that the differences among models became more pronounced as the amount of data decreased and the time of initiation of data collection was delayed.

The usefulness of mathematical models largely depends on how well they can mimic the biological process of milk production [Olori *et al.* 1999]. Selection of the model involves balancing the fitting properties and the requirements for a biological interpretation [Lopez *et al.* 2015]. However, the functions used for lactation curve modelling can represent not only the standard shape of a lactation curve, but several other shapes as well. One possibility is a reversed shape, with an initial decreasing phase to a minimum, followed by an increase until the end of the lactation. Such a shape is typical for fat and protein contents in milk. Another possibility is a continuously increasing or a continuously decreasing curve with no lactation peak. In the case of milk yield such shapes are called atypical curves. Another shape differing from the standard appears for cows calving in autumn in a pasture-based farming system. This shape is characterized by the occurrence of a second lactation peak later during the lactation, and may be explained mainly by seasonal effects [Macciotta *et al.* 2011]. Among all the lactation curve shapes mentioned above, atypical curves are the most frequent next to standard curves. In the literature sources they are reported to occur in

20-30% fitted curves [Olori *et al.* 1999, Rekik and BenGara 2004, Macciotta *et al.* 2005, 2011]. Many studies have shown that the shape of the fitted lactation curve depends on the number of test-day yields and on the combination and distribution of these values along the lactation trajectory. Atypical lactation curves were found less frequently in automatic milking systems than in traditional recording systems (A4 or AT4), in which milk was recorded once a month and peak yield could be easily missed [Macciotta *et al.* 2005, 2011]. For curves with a few parameters, each record influences the whole curve, so atypical curves can be caused by outliers [Druet *et al.* 2003].

The three-parameter Wilmink function is often used in modelling the lactation trajectory, so a deeper analysis and understanding of the significance of parameters for this function may add some interesting elements to the discussion on the occurrence of different shapes of lactation curves for dairy cow milk yields. The objectives of this study were (1) to examine different shapes of lactation curves when the Wilmink function is fitted to test-day yields, and (2) to analyze the relationships between lactation curve shape and factors that cause different shapes of lactation curves in Polish Holstein-Friesian cows.

Material and methods

The data were test-day (TD) milk yields from the first three lactations of 1,359,040 Polish Holstein-Friesian cows registered in the Polish national recording system (SYMLEK). The data were made available by the Polish Federation of Cattle Breeders and Dairy Farmers. There were 9,677,209, 7,201,589, and 4,955,591 TD milk yields from 1,144,721 first, 869,638 second, and 604,602 third lactations, respectively. The cows calved between 1995 and 2008 at the age of 18-48, 29-65 and 41-75 months for the first, second and third time, respectively.

The following restrictions on data were imposed: 1-10 TD records per lactation per cow, first TD before 80 days in milk (DIM), TD yields between 5 and 305 DIM, lactations lasting longer than 99 days, and daily milk yields not exceeding 85 kg. The data had been collected by three testing methods: A4 (56% records), AT4 (26% records) and A8 (18% records). A4 is a standard milk recording method in which test day records are collected twice a day (a.m. and p.m. tests) every four weeks. In the AT4 method, test day records are collected also every four weeks but alternately: a.m. or p.m.. The A8 method involves performing tests twice a day (a.m. and p.m.) but less frequently – every eight weeks. Daily (24 h) milk yields are estimated using two yields (A4 and A8) or from a single yield (AT4) [ICAR 2014]. According to age at calving, the records were divided into five age groups (18-24, 25-26, 27-28, 29-30, 31-48 months), four (29-38, 39-41, 42-44, 45-65 months) and three (41-51, 52-55, 56-75 months) within the first, second and third lactations, respectively. Two seasons of calving were defined: winter (October to March) and summer (April to September). The interval between calving and the first test (in days) was divided into classes called “classes of first test” as follows: up to 19, 20 to 34, 35 to 49, 50 to 64, and 65 to 79 DIM.

A multiple-trait prediction (MTP) procedure was used to fit the lactation curves and estimate partial and 305-d lactation yields from individual TD milk yields [Schaeffer and Jamrozik 1996]. In the MTP method, information concerning standard lactation curves and (co)variances among the parameters of the lactation curve model were incorporated. The parameters of standard lactation curves were estimated within 24 subclasses of lactation by age at calving by season of calving. To estimate the matrix containing variances and covariances among the curve parameters, only cows with first TD before 50 DIM and a minimum 9 TD records per lactation were chosen. Lactation curves were modelled using the Wilmink [1987] function:

$$y=a + b \cdot t + c \cdot e^{-0.05 \cdot t}$$

where t was DIM, a, b and c were parameters to be fitted, and y was milk yield at DIM t. The Wilmink parameters (a, b, c) have biological interpretations related to the shape of the lactation curve: parameter a is associated with the level of production, b with production decrease after peak yield, and c with production increase towards the peak. The value -0.05 is related to the approximate day of peak milk yield [Wilmink 1987].

The different shapes of lactation curves were tested based on analytical properties of the Wilmink function. Depending on the combination of signs of parameters b and c, the Wilmink function described the following curves [Macciotta *et al.* 2005]:

1. a standard lactation curve if $b<0$ and $c<0$,
2. a reversed curve if $b>0$ and $c>0$,
3. a continuously increasing curve if $b>0$ and $c<0$,
4. a continuously decreasing (atypical) curve if $b<0$ and $c>0$.

In our study the Wilmink function classified individual lactation curve shapes essentially into two main groups: standard and atypical. Reversed lactation curves and continuously increasing curves occurred very rarely (4% in the first and 1% in later lactations) and were omitted from the discussion.

Total 305-d milk yield (Milk–305), day of peak milk yield (Peak-D) and milk yield at peak (Peak-M), as well as two persistency measures (PD and b), were calculated. The first measure of persistency (PD) was defined as milk yield at 280 DIM divided by milk yield at 60 DIM, times 100. This measure was selected based on the results of previous research by Otwinowska-Mindur and Ptak [2015] and because it seemed to describe well the potential to maintain a relatively high level of production until the end of the lactation. We chose parameter b of the Wilmink function as the second measure of persistency because of its interpretation as the slope of the lactation curve after peak milk yield. Another reason for choosing b as a persistency measure was its moderate heritability and its ability to be related with reproductive performance and other economically important traits in dairy cows [Muir *et al.* 2004].

Multi-trait analysis of variance for five traits – Milk-305, Peak-D, Peak-M, PD and b – was conducted using the GLM procedure in SAS [SAS Institute 2014]. The effects of factors such as lactation curve shape, season of calving, age of calving class, method of testing, and class of first TD within lactation were examined. The Tukey–Kramer test was used for multiple comparison.

Results and discussion

The three-parameter Wilmink model classified the lactation curve shapes into two main groups: standard and atypical curves. Examples of standard and atypical curves are given in Figure 1, while Table 1 shows characteristics and differences in their distribution. The number of standard patterns was higher in the first lactation (63%); atypical curves were found more often in later lactations (about 50%). Macciotta *et al.* [2005] obtained a similar number of standard lactation curves using the Wilmink model (64%), and a higher number of standard curves using the Wood function (80%). Rekik and BenGara [2004] reported higher frequencies of typical curves than ours (about 75%), but they fitted the Wood function to TD milk yields. They also reported an effect of parity on the occurrence of atypical curves, although the frequencies of atypical lactation curves in later lactations of Holstein-Friesian cows in Tunisia were lower than in the Polish population (22-28%).

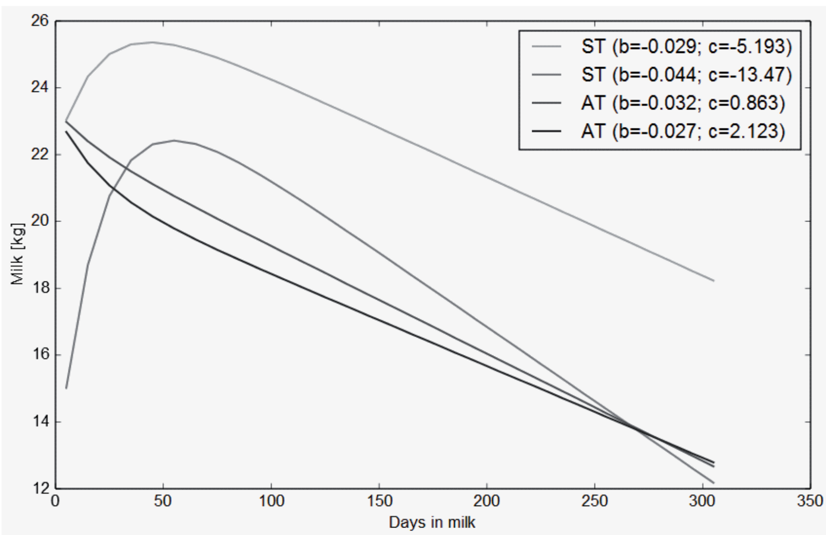

Fig 1. Examples of standard (ST) and atypical (AT) shapes of the first lactation curve.

The correlations among Wilmink model parameters were calculated separately for standard and atypical curves. All these correlations were highly significant (P<0.001) although they differed in value and sign. For standard curves, parameter *a* was negatively correlated with both *b* (-0.75) and *c* (-0.53), while parameters *b* and *c* were positively, but less strongly related to each other (0.41). For atypical curves only parameters *a* and *b* were highly negatively correlated (-0.70). The negative and high correlations between *a* and *b* imply that higher production was associated with a slower decrease of production after the peak. The correlations of *c* with *b* and *a* were close to zero (0.03 and 0.02, respectively) for atypical curves, indicating that parameter *c* was independent of other parameters. Production level (parameter *a*) also had no major influence on the increasing part of the lactation (on parameter *c*). This agrees with Macciotta *et al.* [2005] statement on the independence between the first and second parts of lactation.

Table 1. Characteristics of standard and atypical curves, by parity

Lactation number	Curve shape	No. of lactations		Milk-305[1]		Peak-D[1]		Peak-M[1]		PD[1]		b[1]	
		N[2]	%	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]
1	standard	723,902	63	$5,869^{A}$	1,735	38^{A}	19	23.3^{A}	6.4	64.50^{A}	15.90	-0.0378^{A}	0.0189
	atypical	420,819	37	$4,936^{B}$	1,518	5^{B}	0	22.7^{B}	6.1	68.54^{B}	16.46	-0.0248^{B}	0.0145
2	standard	411,370	47	$6,532^{A}$	2,071	30^{A}	17	28.0^{A}	8.5	53.52^{A}	15.60	-0.0585^{A}	0.0266
	atypical	458,268	53	$5,689^{B}$	1,815	5^{B}	0	28.9^{B}	8.4	59.17^{B}	16.80	-0.0397^{B}	0.0210
3	standard	306,737	51	$6,693^{A}$	2,016	30^{A}	16	29.0^{A}	8.3	51.42^{A}	15.41	-0.0633^{A}	0.0271
	atypical	297,865	49	$5,843^{B}$	1,783	5^{B}	0	30.0^{B}	8.3	56.86^{B}	16.73	-0.0438^{B}	0.0217

[1]Milk-305 is 305-d milk yield (kg); Peak-D is day of peak milk yield; Peak-M is milk yield at peak (kg); PD is persistency calculated as milk yield at 280 DIM divided by milk yield at 60 DIM, and multiplied by 100; b is a parameter of the Wilmink function ($y = a + b \cdot t + c \cdot e^{-0.05 \cdot t}$).
[2]N – number of observations.
[3]SD – standard deviation.
[AB]Values within the same column and lactation number marked by different letters differ significantly at $P<0.01$.

Results of analysis of variance showed that the shape of the lactation curve within each of the first three lactations had a highly significant (P<0.01) effect on all analyzed traits (Tab. 1). Regardless of lactation number, cows with a typical lactation curve shape produced more milk during 305 days than cows with atypical curves. On average, for standard curves the peak occurred at 38 DIM in the first and at 30 DIM in later lactations. All atypical curves were continuously decreasing functions, therefore the day of peak milk yield appeared at the very beginning of the lactation, i.e. at 5 DIM (SD=0). Mean milk yield at peak (about 23 kg in the first and 28-30 kg in later parities) and its standard deviation were similar for the two types of curves. Olori *et al.* [1999] found higher mean milk yield at peak (32 kg in week 7) for first-parity Holstein-Friesian cows in the United Kingdom. Dematawewa *et al.* [2007] also estimated higher peak milk yields (more than 33 kg), occurring much later (93 DIM), during the first lactation.

Two measures of persistency – PD and Wilmink parameter *b* – showed that cows with atypical curves were slightly more persistent than those with standard curves (Tab. 1). Greater persistency was also found for primiparous cows. The above persistency measures could be used alternatively, because they were highly correlated within both standard (0.79) and atypical (0.82) lactation curves. The high correlations were not surprising because of the two measures have similar interpretations: they estimate the slope during the descending phase of lactation.

Table 2 presents the distribution of standard and atypical curves depending on how many days passed from calving to the first test. Almost 90% lactation curves were fitted to the data with the early first test (before 20 DIM), and within each parity there were more curves with standard shapes (55% or more) than with atypical shapes (45% or less). The percentage of atypical curves increased with the distance between calving and the first test. Among lactations with the first test performed after 35 DIM, about 20% of the curves were atypical and only 10-15% were standard.

Generally the frequency of atypical lactation curves decreased with an increasing number of TD records (Tab. 2). If cows had 2-4 TD records, the incidence of atypical curves was highest, but slightly lower in the first lactation (45-53%) than in later ones (61–69%). The percentage of atypical curves was lowest when the lactation curve model was fitted for cows with 8-10 TD yields. It was somewhat surprising that the frequency of atypical curves was low (24%) in the first lactation for cows with only one yield recorded. A more detailed analysis of those data showed that it was not connected with the day of test, because for both types of fitted curves (standard and atypical) yields were recorded between 5 and 79 DIM at similar percentages. It seems that the shapes of the lactation curves of cows with one test depend on the combination of day of test and season of calving, as all atypical curves were fitted for cows that calved in summer (April to September), whereas standard curves were obtained for cows that calved mainly in winter (October to March) (65%). When the only test (for primiparous cows with a standard curve) was performed at 35 DIM or later (up to 79 DIM), a regular incidence was observed for both types of lactation curves. For cows

Table 2. Distributions of standard and atypical lactation curves across different numbers of test-day (TD) records, method of testing, season of calving, and distance between calving and first test, by party

Item	Lactation 1				Lactatation 2				Lactation 3			
	standard		atypical		standard		atypical		standard		atypical	
	number	%	number	%	number	%	number	%	number	%	number	%
Number of TD												
1	1,118	76	350	24	591	46	700	54	449	49	470	51
2	3,404	49	3,612	51	2,101	31	4,683	69	1,810	34	3,538	66
3	7,953	47	8,972	53	4,627	31	10,353	69	4,212	35	7,867	65
4	23,420	55	19,425	45	13,487	35	25,061	65	11,267	39	17,392	61
5	48,165	60	31,453	40	28,389	43	38,172	57	22,586	46	26,016	54
6	38,645	59	27,170	41	21,306	41	30,839	59	16,849	44	21,122	56
7	40,683	57	30,547	43	25,136	41	35,575	59	19,675	45	23,765	55
8	83,851	64	48,105	36	55,387	50	56,277	50	41,731	54	36,189	46
9	125,777	68	60,387	32	79,004	54	66,902	46	58,401	57	43,412	43
10	350,886	65	190,798	35	181,342	49	189,706	51	129,757	52	118,094	48
Method of testing												
A4	413,672	64	231,538	36	236,889	49	247,807	51	175,408	52	159,727	48
AT4	186,827	64	107,088	36	111,951	49	118,784	51	83,431	52	78,332	48
A8	123,403	60	82,193	40	62,530	41	91,677	59	47,898	44	59,806	56
Season of calving												
October – March	482,099	78	134,010	22	285,821	63	125,549	30	208,097	67	98,640	34
April – September	241,803	46	286,809	54	164,860	37	293,408	70	103,986	33	193,879	66
Distance between calving and first test (days)												
5-19	337,780	66	171,105	34	217,193	55	178,178	45	157,381	57	117,145	43
20-34	274,876	62	165,632	38	149,383	44	189,692	56	114,421	49	120,706	51
35-49	58,550	57	44,490	43	25,241	34	49,199	66	19,607	37	32,765	63
50-64	49,907	57	37,792	43	19,084	32	40,400	68	14,992	36	26,679	64
65-80	2,789	61	1,800	39	469	37	799	63	336	37	570	63

that calved in winter the shape of the lactation curve was typical, i.e. with a peak at around 37 DIM on average. For cows that calved in summer the peak occurred very early, at 7 DIM, thus the shape of those curves was very similar to atypical curves (with a peak at 5 DIM).

From the mathematical point of view the parts of a fitted curve before the first data point and after the last one were extrapolated, whereas the part of a fitted curve between the first and last data points was interpolated. When the first test was performed late during lactation (after a peak, in the declining part of the lactation), then the lactation curve was expected to be fitted much better as a descending function than by a curve with a maximum and a standard shape. Such an expectation was supported by Macciotta *et al.* [2005], who wrote that the occurrence of atypical curves was firstly the result of peculiar combinations of TD values and their distribution along the whole lactation, but also the result of the lack of records in the first days after calving.

Table 2 presents the effect of lactation, season of calving and method of testing on the distribution of standard and atypical curves. Generally, fewer atypical curves were observed in the first than in the second and third lactations. The occurrence of curves with an atypical pattern was higher in summer calvings: 54% in the first, 70% in second and 66% in third lactations. These curves were found less frequently when cows calved in winter. There were only 22% first and about 30-34% later lactations with atypical shapes when cows calved between October and March. The effect of calving season on the occurrence of atypical lactation curves was reported by Rekik and BenGara [2004] and Macciotta *et al.* [2005], and this is also in agreement with our results.

About 36% of the first, 51% of the second and 48% of the third lactations had atypical curves when data collected by the A4 and AT4 methods were used (Tab. 2). The occurrence of curves with such a pattern was slightly higher for data collected by the A8 method: 40% for the first, 59% for second and 56% for third lactations.

Table 3 shows the means and standard deviations of Milk-305, Peak-D, Peak-M, PD and *b* by age of calving class and parity. The effect of the age of calving class was significant ($P<0.05$) or highly significant ($P<0.01$) for each trait, except for Milk-305 of cows in the second lactation that calved at ages 39-41 and 42-44 months. For peak day (Peak-D) a significant difference ($P<0.05$) was observed for cows in the second lactation that calved at ages 42-44 and 45-65 months. The PD persistency measure differed significantly ($P<0.05$) for primiparous cows that calved at ages 27-28 and 29-30 months. A significant effect of calving age on milk traits, i.e. peak yield and 305-d lactation yield, was reported by Tekerli *et al.* [2000]. Results of analysis of variance showed that the testing method within lactation had highly significant ($P<0.01$) effects on all analyzed traits (Tab. 4), with one exception: there was no significant difference ($P>0.05$) in the PD measure of persistency in the third lactation when the AT4 and A8 methods were used. Season of calving also had a highly significant ($P<0.01$) effect on Milk-305, Peak-D, Peak-M, PD and *b* within lactation (Tab. 4). The significant influence of the latter on milk traits was demonstrated in studies by Tekerli *et al.* [2000] and Rekik and BenGara [2004]. Tekerli *et al.* [2000] suggested that the relationship between season of calving and peak yield might result from increasing temperature and decreasing fodder, especially during summer. Table 5 presents the effect of interval between calving and first test on five analyzed traits in each of first three parities. In most cases, Milk-305, Peak-D, Peak-M, PD and *b* were affected significantly ($P<0.05$) or highly significantly ($P<0.01$) by the number of days between calving and first test. However, there were some exceptions for each of those traits. For instance, *b* as a measure of persistency did not differ significantly ($P>0.05$) when the distance between calving and first test was longer than one month, i.e. for 3 classes of first test (35-49, 50-64 and 65-80 days).

Table 3. Characteristics of age of calving groups, by parity

Lactation number	Age of calving group (mo.)	No. of lactations		Milk-305[1]		Peak-D[1]		Peak-M[1]		PD[1]		b[1]	
		N[2]	%	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]
1	18-24	278,089	24	5,469^{A}	1,639	26.7^{A}	22	22.7^{A}	6.0	65.60^{A}	15.79	-0.0333^{A}	0.0180
	25-26	295,384	26	5,716^{B}	1,747	28.0^{B}	22	23.6^{B}	6.3	66.16^{B}	16.05	-0.0342^{B}	0.0188
	27-28	219,212	19	5,621^{C}	1,748	26.0^{C}	22	23.4^{C}	6.3	66.66^{aC}	16.27	-0.0328^{C}	0.0187
	29-30	142,782	12	5,492^{D}	1,720	24.5^{D}	22	23.1^{D}	6.3	66.86^{bC}	16.33	-0.0317^{D}	0.0184
	31-48	209,254	18	5,255^{E}	1,709	22.6^{E}	21	22.5^{E}	6.4	64.97^{D}	16.85	-0.0321^{E}	0.0183
2	29-38	294,524	34	5,922^{A}	1,891	16.7^{A}	17	27.7^{A}	8.1	55.60^{A}	16.13	-0.0486^{A}	0.0249
	39-41	218,536	25	6,217^{B}	2,015	16.6^{B}	17	29.1^{B}	8.6	56.75^{B}	16.52	-0.0494^{B}	0.0264
	42-44	155,298	18	6,230^{B}	2,021	17.1^{aC}	17	28.9^{C}	8.6	57.43^{C}	16.61	-0.0485^{C}	0.0261
	45-65	201,280	23	6,079^{C}	2,041	17.2^{bC}	18	28.4^{D}	8.6	56.82^{D}	16.79	-0.0477^{D}	0.0251
3	41-51	211,145	35	6,130^{A}	1,822	16.6^{A}	16	29.2^{A}	8.0	52.36^{A}	15.77	-0.0550^{A}	0.0258
	52-55	164,688	27	6,385^{B}	1,982	17.3^{B}	17	30.0^{B}	8.4	54.55^{B}	16.34	-0.0540^{B}	0.0272
	56-75	228,769	38	6,326^{C}	2,036	18.6^{C}	18	29.4^{C}	8.5	55.38^{C}	16.61	-0.0522^{C}	0.0265

[1]Milk-305 is 305-d milk yield (kg); Peak-D is day of peak milk yield; Peak-M is milk yield at peak (kg); PD is persistency calculated as milk yield at 280 DIM divided by milk yield at 60 DIM, and multiplied by 100; b is a parameter of the Wilmink function ($y = a + b \cdot t + c \cdot e^{-0.05 \cdot t}$).
[2]N – number of observations.
[3]SD – standard deviation.
[aA...]Values within the same column and lactation number marked by different letters differ significantly at: small letters – $P<0.05$; capitals – $P<0.01$.

The results of this study showed that the Wilmink model produces two main groups of lactation curves: standard and atypical. Atypical curves were found more often in later rather than in the first lactations. Milk yield recording should start as early as possible, because the distance between the first test and calving is a factor significantly affecting the occurrence of atypical lactation curves. Additionally, season and parity

Table 4. Characteristics of methods of testing and seasons of calving by parity

Lactation number	Item	No. of lactations		Milk-305[1]		Peak-D[1]		Peak-M[1]		PD[1]		b[1]	
		N[2]	%	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]
	method of testing												
1	A4	645,210	56	$5,539^{A}$	1,855	26.4^{A}	22	23.2^{A}	6.6	65.07^{A}	16.77	-0.0338^{A}	0.0189
	AT4	293,915	26	$5,792^{B}$	1,602	27.5^{B}	23	24.0^{B}	6.1	68.03^{B}	15.59	-0.0328^{B}	0.0189
	A8	205,596	18	$5,103^{C}$	1,302	22.0^{C}	19	21.5^{C}	5.1	65.98^{C}	15.09	-0.0308^{C}	0.0163
2	A4	484,696	56	$6,105^{A}$	2,142	17.3^{A}	17	28.6^{A}	8.9	55.06^{A}	16.82	-0.0505^{A}	0.0265
	AT4	230,735	27	$6,373^{B}$	1,870	18.2^{B}	18	29.6^{B}	8.3	58.38^{B}	16.19	-0.0487^{B}	0.0259
	A8	154,207	18	$5,606^{C}$	1,492	13.5^{C}	15	26.3^{C}	6.8	58.20^{C}	15.35	-0.0425^{C}	0.0208
3	A4	335,135	55	$6,246^{A}$	2,104	18.1^{A}	17	29.4^{A}	8.7	52.67^{A}	16.67	-0.0553^{A}	0.0274
	AT4	161,763	27	$6,576^{B}$	1,842	18.7^{B}	18	30.7^{B}	8.1	55.87^{B}	16.01	-0.0540^{B}	0.0267
	A8	107,704	18	$5,906^{C}$	1,497	14.4^{C}	15	27.8^{C}	6.9	55.91^{B}	15.09	-0.0480^{C}	0.0219
	season of calving												
1	October - March	616,109	54	$5,671^{A}$	1,717	31.6^{A}	21	23.3^{A}	6.2	63.83^{A}	15.60	-0.0369^{A}	0.0185
	April - September	528,612	46	$5,356^{B}$	1,704	19.2^{B}	21	22.9^{B}	6.3	68.51^{B}	16.57	-0.0284^{B}	0.0173
2	October - March	450,681	52	$6,224^{A}$	2,004	21.1^{A}	18	28.2^{A}	8.4	54.18^{A}	15.47	-0.0538^{A}	0.0258
	April - September	418,957	48	$5,941^{B}$	1,954	12.3^{B}	15	28.8^{B}	8.5	58.99^{B}	17.17	-0.0430^{B}	0.0241
3	October - March	312,083	52	$6,415^{A}$	1,977	21.8^{A}	18	29.3^{A}	8.3	52.03^{A}	15.27	-0.0588^{A}	0.0267
	April - September	292,519	48	$6,123^{B}$	1,913	13.0^{B}	15	29.8^{B}	8.4	56.30^{B}	17.06	-0.0482^{B}	0.0251

[1]Milk-305 is 305-d milk yield (kg); Peak-D is day of peak milk yield; Peak-M is milk yield at peak (kg); PD is persistency calculated as milk yield at 280 DIM divided by milk yield at 60 DIM, and multiplied by 100; b is a parameter of the Wilmink function ($y = a + b \cdot t + c \cdot e^{-0.05 \cdot t}$).
[2]N – number of observations.
[3]SD – standard deviation.
[ABC]Values within the same item (testing method or season of calving) and lactation number marked by different letters differ significantly at $P<0.01$.

Table 5. Characteristics of intervals between calving and first test, by parity

Lactation number	Interval between calving and first test (day)	No. of lactations		Milk-305[1]		Peak-D[1]		Peak-M[1]		PD[1]		b[1]	
		N[2]	%	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]	mean	SD[3]
1	5-19	508,885	44.5	$5,567^{aA}$	1,742	28.8^{A}	23	23.2^{A}	6.1	65.86^{A}	16.36	-0.0336^{A}	0.0189
	20-34	440,508	38.5	$5,577^{bA}$	1,743	25.1^{B}	22	23.4^{B}	6.4	65.90^{AB}	16.24	-0.0332^{B}	0.0184
	35-49	103,040	9.0	$5,359^{B}$	1,592	20.9^{C}	19	22.5^{C}	6.2	66.74^{C}	15.82	-0.0310^{C}	0.0174
	50-64	87,699	7.7	$5,232^{C}$	1,548	19.5^{D}	18	21.9^{D}	6.1	66.27^{D}	15.81	-0.0308^{C}	0.0172
	65-80	4,589	0.4	$5,323^{B}$	1,804	21.0^{C}	19	22.0^{D}	6.9	66.45^{ABCD}	16.31	-0.0309^{C}	0.0181
2	5-19	395,371	45.5	$6,110^{A}$	2,009	20.2^{A}	19	28.0^{A}	8.1	56.31^{A}	16.64	-0.0496^{A}	0.0263
	20-34	339,075	39.0	$6,139^{aB}$	2,022	15.3^{B}	16	29.1^{B}	8.8	56.06^{B}	16.40	-0.0492^{B}	0.0255
	35-49	74,440	8.6	$5,898^{C}$	1,815	11.5^{C}	13	28.2^{C}	8.5	58.04^{C}	16.13	-0.0444^{C}	0.0231
	50-64	59,484	6.8	$5,885^{DCE}$	1,792	10.4^{D}	11	28.0^{AD}	8.5	58.26^{C}	16.11	-0.0441^{C}	0.0230
	65-80	1,268	0.1	$5,981^{bABCE}$	1,899	11.1^{CD}	12	27.9^{ACD}	8.7	59.81^{D}	16.84	-0.0432^{C}	0.0248
3	5-19	274,526	45.4	$6,282^{A}$	1,974	20.6^{A}	18	29.1^{A}	8.0	53.84^{A}	16.44	-0.0546^{A}	0.0272
	20-34	235,127	38.9	$6,320^{B}$	1,986	16.5^{B}	16	30.0^{B}	8.7	53.74^{A}	16.24	-0.0543^{B}	0.0264
	35-49	52,372	8.7	$6,149^{C}$	1,797	12.2^{C}	13	29.5^{C}	8.4	55.66^{B}	15.98	-0.0497^{C}	0.0242
	50-64	41,671	6.9	$6,119^{C}$	1,771	11.0^{D}	12	29.3^{D}	8.4	55.83^{B}	15.90	-0.0493^{C}	0.0240
	65-80	906	0.1	$6,176^{ABC}$	1,942	11.3^{CD}	12	28.9^{ACD}	8.8	57.09^{B}	16.13	-0.0482^{C}	0.0246

[1]Milk-305 is 305-d milk yield (kg); Peak-D is day of peak milk yield; Peak-M is milk yield at peak (kg); PD is persistency calculated as milk yield at 280 DIM divided by milk yield at 60 DIM, and multiplied by 100; b is a parameter of the Wilmink function ($y = a + b \cdot t + c \cdot e^{-0.05 \cdot t}$).
[2]N – number of observations.
[3]SD – standard deviation.
[aA...]Values within the same column and lactation number marked by different letters differ significantly at: small letters – $P<0.05$; capitals – $P<0.01$.

also had an effect on the occurrence of atypical curves in the Polish Holstein-Friesian population. The effect of season was particularly conspicuous for cows that calved between April and September with only one yield recorded.

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